The fracture behaviour of notched specimens of polymethylmethacrylate

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The fracture behaviour of notched specimens of polymethylmethacrylate has been examined for a wide range of geometries in Charpy impact tests, and in tensile and slow bend fracture tests. It was found that the failure of the very sharply notched specimens was consistent with linear elastic fracture mechanics and defined a constant fracture toughness K_{IC} for a constant notch tip radius, whereas the blunt notched specimens failed at a constant critical stress at the root of the notch.

1. Introduction

Most studies of the fracture behaviour of glassy polymers have followed the linear elastic fracture mechanics approach, in terms of either a surface energy, in the pioneering research of Benbow and Roesler [1] and Berry [2], or the Irwin stress intensity factor [3, 4]. Although the work in this area has been primarily concerned with the growth of slow cracks, recent studies by Brown [5] and by Williams *et al.* [6] have extended the linear fracture mechanics to impact tests on sharply notched specimens.

In a very different approach, Vincent [7] has discussed the fracture behaviour of polymers in terms of brittle/ductile transitions, and following the classical proposals of Orowan [8], has drawn a similar distinction to the latter regarding notched brittleness. Essentially it is assumed that whereas notching changes the effective yield stress, the brittle strength is unaltered, and has meaning as a critical parameter in fracture tests, including notched fracture tests. This view has received support from recent results of Gotham [9], who showed that the tensile fracture of notched samples of polymethylmethacrylate was consistent with a constant fracture stress calculated on the basis of the Neuber stress concentration factor [10] for the particular notch shape employed.

In previous studies of the fracture behaviour of polyethylene terephthalate both types of approach have appeared to have relevance. In the earliest broad survey of failure in this polymer by Stearne and Ward [11], brittle/ductile transitions were observed, and both the influence of structural factors such as crystallinity and molecular weight and notch sensitivity could be very well explained qualitatively in terms of the Vincent/Orowan ideas. On the other hand, the behaviour of sharply notched tensile and cleavage specimens in fracture was consistent with linear elastic fracture mechanics [12], and an empirical correlation was observed between impact strength and fracture toughness for a wide range of amorphous samples of different molecular weight.

In view of the importance of practical impact tests such as the Charpy and Izod notched impact tests, it seemed valuable to make a detailed study of the impact fracture behaviour of a wide range of notched specimens, particularly including the comparatively blunt notches similar to those employed in the practical tests. For completeness, similar specimens were also broken in tensile and slow bend tests. We have carried out these experiments on polymethylmethacrylate primarily because of the ready availability of standard material.

It will be shown that the failure of all the bluntly notched specimens is consistent with a constant failure stress rather than a constant fracture toughness. The implications of these results for the general failure testing of plastics will be discussed.

2. Experimental

Impact bend, four-point bend and uniaxial tensile fracture tests were undertaken on a wide range of notched specimens of polymethylmethacrylate (PMMA).

2.1. Preparation of specimens
All the experimental work was carried out on a commercial grade of PMMA manufactured by ICI Ltd, Perspex cast sheet of nominal thickness 0.3 cm.

The impact and four-point bend specimens were all 5 cm in length with widths varying from 0.6 to 1.2 cm. The tensile fracture specimens were 15 cm in length and 5 cm wide, some with a reduction to 3.5 cm width in a test length of 6 cm.

All specimens were notched, the bending specimens on one side and the tensile specimens on both sides, at the middle point of their length and two basic techniques were used. First, to obtain very sharp notches a razor blade was pushed slowly into the centre of the side of the specimen, until the crack, which travelled slowly in from the razor blade reached a length of about 3 mm. Secondly, to obtain a more bluntly notched specimen, holes were drilled through the specimen and a jeweller's saw used to cut through from the edge of the specimen to the hole. Both these techniques resulted in cracks with 0° flank angles. Peterson [13] has shown that flank angles from 0° to 45° change the stress concentration factor at the root of a notch by only very small amounts, which have been neglected in the present work. Since the size of a drilled hole is not necessarily the size of the drill used, particularly in the case of small holes, all the specimens were measured for notch length and notch tip radius after manufacture, using a travelling microscope.

Finally, some specimens were annealed for 1 h at 108°C after manufacture to examine the possibility that internal stresses caused by machining could modify the fracture behaviour. In general, however, the specimens were not annealed.

2.2. Impact tests
The impact tests were carried out on a Hounsfield Plastics Impact Tester, which is a Charpy machine, supplied with striking tups of various weights to cover a wide range of fracture energies, and calibration charts to furnish the actual value of energy obtained. The tups have a striking speed of 2.54 m sec^{-1}.

The notch tip radii for the impact test specimens varied from 0.122 to 0.03 cm and razor notches, all with $0 < a/W < 0.6$ where a, W are the notch depth and the width of the specimen respectively. Series of specimens with varying notch lengths for each particular value of notch tip radius were used throughout the tests.

2.3. Four-point bend tests
The four-point bend tests were undertaken on an Instron tensile testing machine, with a special set of grips which loaded the specimens in four-point bending in an identical geometrical manner to that obtained in the Hounsfield Impact Tester. A cross-head speed of 0.2 cm min^{-1} was used, the $\frac{1}{4}$ sec pen recorder being adequate in response to provide an accurate recording of the load as a function of time. Single edge notched specimens were tested, with notch dimensions in a comparable range to those described for the impact tests.

2.4. Uniaxial tensile fracture tests
The tensile fracture tests were carried out on an Instron tensile testing machine, with a special set of grips. In these grips, the flat tensile specimens were clamped at their ends, the clamps ensuring that the load was spread uniformly across the cross-section. These clamps were held in universal joints to allow side movement and hence reduce to a negligible amount the bending stresses which could be caused by rigid mounting rods with even small misalignments.

A cross-head speed of 10 cm min^{-1} was employed. To obtain a sufficiently fast response, the load cell was powered and monitored by a Sangamo Transducer meter whose output was fed to an ultra-violet light recorder. The system was dead weight calibrated.

In the tensile fracture tests, double edge notched specimens were used with notch tip radii in the range 0.150 to 0.028 cm and razor notches with $0 < a/W < 0.8$. (The larger range of a/W is allowed, because the stress concentration factor calculations are valid for all a/W, although the stress intensity factor calculations, used for the compliance measurements of bending specimens, are only valid in the narrower range). For ease of manufacture, double edge razor notched specimens were limited to $a/W \simeq 0.4$, and a small razor notch was introduced at the base of a 0.0025 cm wide slot cut with a slitting saw.

3. Theory
In this paper we are concerned with two approaches to the failure of notched specimens: firstly linear elastic fracture mechanics where we seek to calculate the critical stress *intensity* factor, or fracture toughness K_{IC}, and secondly

the constant fracture stress approach. In this second case, we require the stress *concentration* factor for each test geometry, in order to calculate the stress at the root of the notch; it is this stress which is now the critical parameter.

3.1. Tensile fracture

For tensile fracture, the fracture toughness K_{IC} was calculated from the relationship [14]

$$K_{IC} = \sigma a^{\frac{1}{2}} \left[1.98 + 0.36\left(\frac{2a}{W}\right) - 2.12\left(\frac{2a}{W}\right)^2 + 3.42\left(\frac{2a}{W}\right)^3\right]$$

where σ is the failure stress calculated on the cross-sectional area, W is the width of the doubly notched specimen and a the depth of the notch.

The stress concentration factor for a particular notch configuration was taken from Peterson's calculations ([13], p. 25 f.f.) Peterson has used Neuber's [10] results to plot convenient graphs from which stress concentration factors were directly obtained knowing the individual specimen dimensions, the original work of Neuber having employed a directly theoretical approach using his own modification of the three-dimensional Stress Function method.

3.2. Impact and slow bend tests

The analysis of the impact and slow bend tests is identical, as in both cases specimens of identical configuration are fractured in four-point bending. The only difference between the two tests is in the speed of testing.

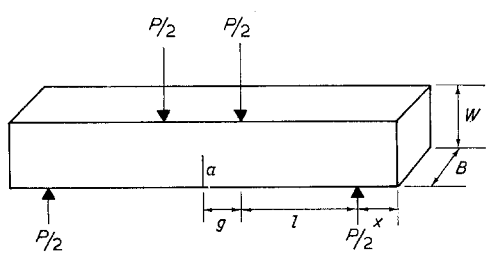

Figure 1 Schematic diagram of four-point bend test.

The starting point for the analysis is to take the result obtained by Brown [5] for the compliance C (relative deflection of the loading points per unit load) of a cracked beam in four point bending. We have

$$C = \frac{18l^2}{E^*BW^2} f\left(\frac{a}{W}\right) + \frac{l}{2E^*BW^3}[4l(3g + l) + 3W^2(1 + \nu)]$$

where E^* is the reduced modulus, equal to Young's modulus E, in condition of plane stress, and to $E/(1 - \nu^2)$ in plane strain (ν is Poisson's ratio), a, W and B are the depth of the crack, the width and breadth of the specimen respectively, l and g are apparatus dimensions (see Fig. 1) and $f(a/W)$ is an integrated function of a/W.

Assuming that we have a linear elastic totally brittle material (which will be shown to be valid for PMMA under the condition of testing used here) the elastically stored energy in the specimen immediately prior to failure is

$$U_0 = \tfrac{1}{2}P_0^2C$$

where P_0 is the load immediately prior to failure. It is this quantity U_0 which the impact test seeks to measure.

A correction for the kinetic energy of the specimen itself immediately prior to fracture was calculated (see Appendix) and has been shown to be a realistic estimate of the difference found between experimentally measured energy to fracture and theoretically calculated energy to fracture. This correction factor was then subtracted from measured fracture energy systematically prior to mathematical manipulation and hence will not be referred to further in this paper.

On the basis that fracture relates to a critical strain energy release rate the critical stress intensity factor or fracture toughness K_{IC} is then given by the Irwin-Kies relationship [15]

$$K_{IC}^2 = \frac{P_0^2}{2B} E^* \frac{dC}{da}\,.$$

If, on the other hand, we assume that fracture relates to a critical stress at the root of the notch we argue as follows: the bending moment M applied to a specimen in four-point bending is given by

$$M = \frac{Pl}{2}$$

where P is the applied load and l is an apparatus dimension. Hence the elastically stored energy immediately prior to fracture may be written as

$$U_0 = \tfrac{1}{2}\left(\frac{2M_0}{l}\right)^2 C$$

where M_0 is the applied bending moment immediately prior to fracture. Therefore, we have

$$M_0 = \frac{l}{2}\sqrt{\left(\frac{2U_0}{C}\right)}.$$

Hence we may now calculate the applied moment necessary to initiate fracture in a specimen, from the energy to fracture and the geometrical arrangements. If it is assumed that the compliance of a specimen varies only with crack length and not with crack tip radius and flank angle (for flank angles less than 45°), which again will be shown to be valid for the wide range of geometries used, then it is possible to use this calculated bending moment, together with a calculated stress concentration factor, to calculate the maximum stress at the root of the notch at the point of fracture (σ_m). The stress concentration factors used were taken directly from Neuber's calculations (using Fig. 104, p. 181). The nominal stress σ_n for the bending case is given in terms of the bending moment M by the relationship

$$\sigma_n = \frac{6M}{B(W-a)^2}$$

as quoted by Neuber as his "elementary bending stress" (Equation 111, p. 51). Hence calculating the maximum value of σ_n by substituting M_0 into this equation, and multiplying this by the geometrical stress concentration factor will yield the required value of σ_m.

4. Results and discussion

4.1. Preliminary observations

The results obtained on the Instron Tensile Testing machine for both direct tensile tests and four-point bend tests on razor and radiussed notched specimens produced linear stress/displacement plots to the point of fracture. This confirms that it is valid to assume that the material, under the conditions of test obtained here, is linear elastic and brittle.

The slow bend tests were initially analysed to obtain the calculated compliances. The results shown in Fig. 2 confirm that the calculated and measured compliances are in good correspondence. The slope of the best fit straight line gives a Young's modulus value for PMMA of 3.6×10^9 N m^{-2} and a machine compliance of 0.2×10^{-6} m N^{-1}.

Samples with a notch tip radius of 0.05 cm were also annealed prior to testing, as described above. The results show well both that the

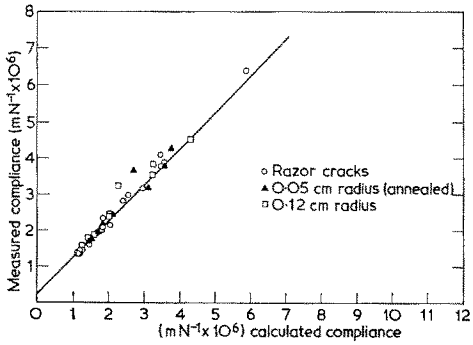

Figure 2 Measured compliance versus calculated compliance for slow bend test.

compliance calculation is valid for the wide range of notch dimensions used, and also that annealing does not effect the stiffness of PMMA.

4.2. Fracture criteria

We will seek to show that the results are consistent with there being two different situations: the blunt notch situation where the maximum stress determines fracture, and the very sharp notch situation where linear elastic fracture mechanics applies.

4.2.1. The blunt notch situation

Here it is proposed that the strength, either in an impact test, or in a slow bend test, or in a uniaxial tensile test, is a measure of the difficulty of forming the initial craze and crack on the surface of the specimen. This has been previously proposed for the low temperature fracture of unnotched specimens of amorphous polyethylene terephthalate [12], and for the low temperature fracture of polystyrene [16]. Once the crack is formed there is always sufficient energy for crack propagation, i.e. the stored elastic energy immediately prior to fracture is greater than that required for crack propagation. Hence if it is attempted to calculate the fracture toughness from the stored elastic energy, too high a value is obtained. The strength of the blunt notched specimen is determined by the maximum stress at the tip of the notch and we propose that this stress is that necessary to cause a tensile craze to occur in the material at this point at the relevant strain rate. The maximum stress at the root of the notch is, therefore, a necessary and sufficient condition for failure, and the product of the applied load/net cross-sectional area (the nominal stress) at failure and the stress concentration factor is a constant. Gotham has expressed this condition in a rather different manner by showing that the nominal stress at failure is proportional to the reciprocal of the stress concentration factor. In Figs. 3, 4 and 5 results for the impact tests, the slow bend tests and the tensile fracture tests are shown. Fig.

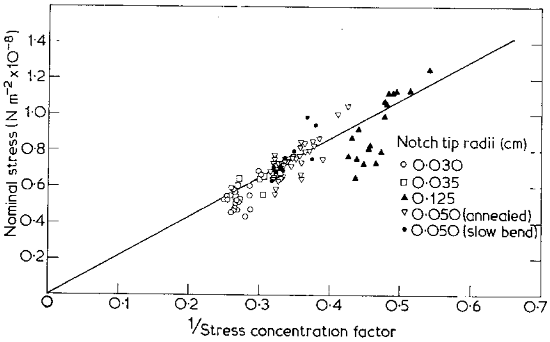

Figure 3 Nominal stress versus inverse stress concentration factor for impact bend tests and slow bend tests.

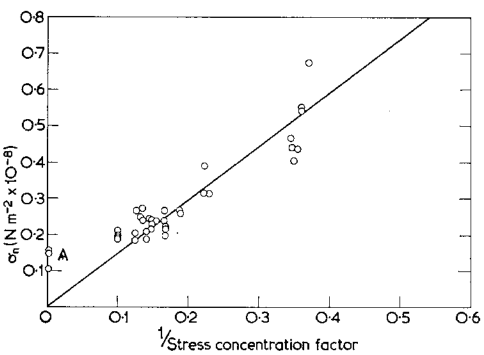

Figure 4 Nominal stress (on reduced cross sectional area) versus inverse stress concentration factor for notched tensile tests.

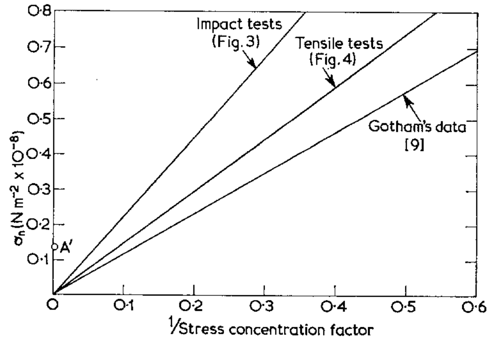

Figure 5 Comparison of results from impact tests and notched tensile tests.

5 shows that there is good agreement with Gotham's conclusion that the tensile fracture of notched specimens of PMMA is in accordance with a critical stress for fracture. Moreover, the results of Fig. 3 show that this also applies to impact tests and slow bend tests. This result is of considerable technological importance, as the blunt notched Charpy test is used as a practical test of fracture toughness.

The spread of results is not great considering the very wide range of notch tip radii and lengths. Moreover, the annealed samples clearly fall in the same family of results. The failure stress indicated by the best line through the impact test points is 2.2×10^8 N m^{-2} and compares well with Gotham's value of 1.1×10^8 N m^{-2} bearing in mind that Gotham used a cross-head speed of 5.0 cm min^{-1} and that the impact striking speed is $\sim$ 250 cm sec^{-1}. The tensile tests suggest a fracture stress of 1.5×10^8 N m^{-2} for the cross-head speed of 10 cm m^{-1}.

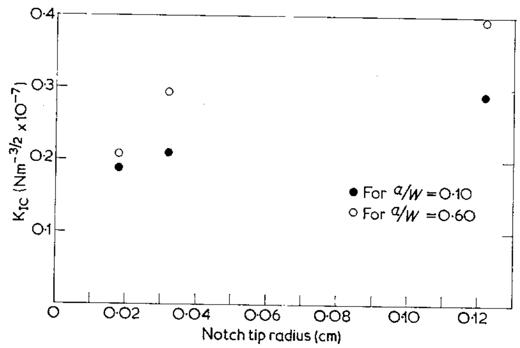

Figure 6 Measured K_{IC} versus notch tip radius for impact bend tests.

These results can also be analysed using the fracture mechanics approach. Fig. 6 shows that there is a consistent increase in apparent toughness with notch tip radius. Moreover, there is a variation of apparent toughness K_{IC} with notch length for blunt notched specimens. This is consistent with the view that has been expressed, namely that the necessary condition for failure to initiate is that the craze stress be achieved, and that with blunter notches increasing elastic energy must be supplied to satisfy this condition.

4.2.2. The very sharp notch situation

In the case of the razor notched specimens, the stress criterion for craze formation is not important. The very sharp razor has already produced a crack with a craze and the craze is of such a length that it cancels the stress singularity at the crack tip. Related studies of fatigue fracture in glassy polymers [17] has shown that under applied load the crack grows to a length determined by the applied stress and the need to cancel the stress singularity, i.e. the craze length is determined by the stress intensity factor even although this does not rise to its critical value for fracture to occur. In this sharp crack situation, then, the rate of energy release defines a necessary and sufficient condition for fracture and the stress intensity factor for fracture is a constant.

It should perhaps be emphasized that we are not suggesting that the fracture behaviour becomes independent of notch tip radius for very sharp notches, but that the fracture behaviour can be represented by a constant fracture toughness K_{IC} for a consistently manufactured very sharp notch, i.e. for constant notch tip radius.

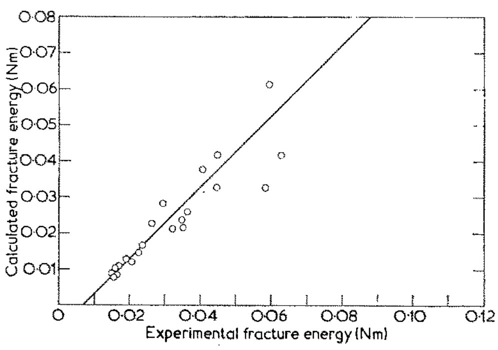

Figure 7 Calculated versus experimental fracture energy for sharp notch impact bend tests.

The razor notched impact bend tests were analysed following the method of Brown [5], plotting the calculated energy to fracture as a function of the measured energy to fracture. In this case the good straight line relationship (Fig. 7) shows a constant critical stress intensity factor of 0.17×10^7 N m$^{-3/2}$. If we attempt to analyse these results in terms of a critical fracture stress (points A A′ on Figs. 4 and 5) we see that the failure stress is too high for the very high stress concentration factor. On our explanation of these results, this result arises because the very sharp notch has already allowed the craze stress to be reached, at its tip, at a stress less than the applied failure stress of the specimen. This condition is, however, only a necessary and not a (necessary and) sufficient condition for fracture. We must still supply stored elastic energy until the rate of release of this energy is sufficient to allow propagation of the fracture.

5. Conclusion

To cause failure, a crack must both be initiated and propagated and, depending on the geometry of the particular test, which ever process requires the greater energy input will be the fracture governing parameter.

For sharp notches, conditions for initiation of a crack/craze are satisfied before the energy imbalance required to propagate it, and hence the latter is of prime importance and the test is characterized by the fracture toughness K_{IC}. However, for blunt notches, the elastically stored energy, required to initiate the crack by producing the craze (i.e. the deformation needed to meet the critical stress at the notch tip) is sufficiently large for the subsequent rate of release of stored energy to be greater than that required for a stable moving crack. The crack therefore accelerates and catastrophic failure occurs.

It is necessary, therefore, not only for impact testing, but also in other forms of failure testing, to ensure that the correct interpretation is made of measured data, bearing in mind the results presented here. Recent work by Vincent [18] has shown that for the failure of ductile thermoplastics, the criterion of a critical value of the stress intensity factor is not valid. However, Ferguson and Williams [19] have found that the Dugdale model is still useful, and have shown that it can produce a critical Crack Opening Displacement criterion for failure. It is important to note that the failure of the stress intensity factor criterion for blunt notched specimens of PMMA is not capable of this explanation. In PMMA, the plastic zone (i.e. the craze zone) is always very small and the fracture toughness criterion, when it is applicable, is exactly equivalent to a critical Crack Opening Displacement criterion at a constant craze stress. (For further discussion of this point see [17] and [20].)

Appendix

In order to calculate the kinetic energy of an impact bend specimen it is assumed that at the point of catastrophic failure, the specimen halves are thin bars rotating about their outer support points, with the inner (striking) points moving with the same velocity (V) as the striking tup.

If we consider one half of the specimen, as shown in Fig. 1, and take an element, thickness dy, a distance y from the outer support, the velocity of this element is Vy/l.

The mass of the element is $WB\mathrm{d}y\rho$, where ρ is the density of the material. Hence the kinetic energy of the element is:

$$\tfrac{1}{2}WB\mathrm{d}y\rho\left(\frac{Vy}{l}\right)^2 .$$

Therefore, the kinetic energy of the whole specimen is:

$$WB\rho\,\frac{V^2}{l^2}\int_{-x}^{l+g} y^2\,\mathrm{d}y\,.$$

The results shown in Fig. 7, are from a series of razor notched impact bend tests, with a constant specimen width of 1 cm. Following the method of Brown [5] the theoretically calculated fracture energy is plotted against the experimentally found fracture energy. Taking the best fit straight line, an intercept of 0.007 Nm is found on the experimentally determined energy axis and this compares well with the value for the kinetic energy of such a size specimen of 0.006 Nm, calculated using the above equation.

Acknowledgements

We would like to thank Dr R. A. Duckett for his help and much useful discussion throughout the preparation of this report. This work has been carried out with the support of Procurement Executive, Ministry of Defence.

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Received 25 March and accepted 24 May 1974.